UNIVERIDAD DE EL SALVADOR

FACULTAD MULTIDISCIPLINARIA DE OCCIDENTE

DEPARTAMENTO DE INGENIERÍA Y ARQUITECTURA

INGENIERÍA DE SISTEMAS INFORMÁTICOS

MATERIA: ANÁLISIS NUMÉRICO

GUÍA DE EJERCICIOS

"PROBLEMAS DE VALOR INICIAL PARA ECUACIONES DIFERENCIALES ORDINARIAS CON EL MÉTODO DE TAYLOR"

1. Resuelve el siguiente problema de valor inicial utilizando el método de Taylor de orden 2 para encontrar una aproximación de $y(0.2)$ con $h = 0.1$.

$$y' = x^2 - y, y(0) = 1.$$

- $f(x,y) = x^2 - y$
- $f_x = 2x$
- $f_y = -1$
- $y'' = f_x + f_y \cdot f = 2x - x^2 + y$

$Paso$ 0: $x_0 = 0, y_0 = 1$

- $f_0 = 0^2 - 1 = -1$
- $y_0'' = 2 \cdot 0 - 0^2 + 1 = 1$
- $y_1 = y_0 + h \cdot f_0 + \frac{h^2}{2} \cdot y_0''$
- $= 1 + 0.1(-1) + \frac{0.01}{2}(1)$
- $= 0.905$

$Paso$ 1: $x_1 = 0.1, y_1 = 0.905$

- $f_1 = 0.1^2 - 0.905 = -0.895$
- $y_1'' = 2 \cdot 0.1 - 0.1^2 + 0.905 = 1.095$
- $y_2 = y_1 + 0.1(-0.895) + \frac{0.01}{2}(1.095)$

$$\rightarrow y(0.2) \approx 0.82098$$

2. Aproximar el valor de $y(0.3)$ aplicando el método de Taylor de orden 3 con $h = 0.1$.

$$y^{'} = y - x^2 + 1, y(0) = 0.5.$$

- $f = y - x^2 + 1$
- $f_x = -2x$
- $f_y = 1$
- $y^{''} = f_x + f_y \cdot f = -2x + y - x^2 + 1$
- $\frac{\partial y^{''}}{\partial x} = -2 - 2x$
- $\frac{\partial y^{''}}{\partial y} = 1$
- $y^{'''} = (-2 - 2x) + f = y - x^2 - 2x - 1$
- $y_{n+1} = y_n + h \cdot f + \frac{h^2}{2} \cdot y^{''} + \frac{h^3}{6} \cdot y^{'''}$

Paso 0: $x0 = 0, y0 = 0.5$

- $f_0 = 1.5$
- $y_0^{''} = 1.5$
- $y_0^{'''} = -0.5$
- $y_1 \approx 0.65742$

Paso 1: $x1 = 0.1, y1 \approx 0.65742$

- $f_1 \approx 1.64742$
- $y_1^{''} \approx 1.44742$
- $y_1^{'''} \approx -0.55258$
- $y_2 \approx 0.829307$

Paso 2: $x2 = 0.2, y2 \approx 0.829307$

- $f_2 \approx 1.789307$
- $y_2^{''} \approx 1.389307$
- $y_2^{'''} \approx -0.610693$
- $y_3 \approx 1.01508$

$$\rightarrow y(0.3) \approx 1.01508$$

3. Emplear el método de Taylor de orden 2 para hallar una aproximación de $y(0.4)$ en pasos

   de $h = 0.2$.

$$y' = y\cos x,\ y(0) = 1.$$

- $f = y \cdot \cos x$
- $f_x = -y \cdot \sin x$
- $f_y = \cos x$
- $y'' = f_x + f_y \cdot f = y(-\sin x + \cos^2 x)$

Paso 0: $x0 = 0, y0 = 1$

- $f_0 = 1$
- $y_0'' = 1$
- $y_1 = 1 + 0.2 \cdot 1 + \frac{0.04}{2} \cdot 1 = 1.22$

Paso 1: $x1 = 0.2, y1 = 1.22$

- $\cos 0.2 \approx 0.980$
- $\sin 0.2 \approx 0.19867$
- $f_1 \approx 1.19549$
- $y_1'' \approx 0.93048$
- $y_2 = 1.22 + 0.2 \cdot 1.19549 + \frac{0.04}{2} \cdot 0.93048$
- ≈ 1.47773

$$\rightarrow y(0.4) \approx 1.47773$$

4. Usa el método de Taylor de tercer orden para aproximar $y(0.2)$ con $h = 0.1$.

$$y' = e^x - y, y(0) = 1.$$

- $f = e^x - y$
- $f_x = e^x$
- $f_y = -1$
- $y'' = f_x + f_y \cdot f = e^x - (e^x - y) = y$
- $y''' = \frac{d}{dx}(y) = y' = e^x - y$

Paso: $x0 = 0, y0 = 1$

- $f_0 = 0$
- $y_0'' = 1$
- $y_0''' = 0$
- $y_1 = 1.005$

Paso 1: $x1 = 0.1, y1 = 1.005$

- $e^{0.1} \approx 1.10517$
- $f_1 \approx 0.10017$
- $y_1'' = 1.005$
- $y_1''' \approx 0.10017$
- $y_2 \approx 1.02006$

$$\rightarrow y(0.2) \approx 1.02006$$

5. Con el método de Taylor de orden 2, estimar el valor de $y(0.3)$ en pasos de $h = 0.1$.

$$y' = x + y, y(0) = 0.$$

- $f = x + y$
- $f_x = 1$
- $f_y = 1$
- $y'' = 1 + x + y$

Paso 0: $x0 = 0, y0 = 0$

- $f_0 = 0$
- $y''_0 = 1$
- $y_1 = 0.005$

Paso 1: $x1 = 0.1, y1 = 0.005$

- $f_1 = 0.105$
- $y''_1 = 1.105$
- $y_2 \approx 0.021025$

Paso 2: $x2 = 0.2, y2 \approx 0.021025$

- $f_2 \approx 0.221025$
- $y''_2 \approx 1.221025$
- $y_3 \approx 0.049233$

$$\rightarrow y(0.3) \approx 0.04923$$

6. Aplicar el método de Taylor de orden 3 para aproximar $y(0.3)$, con paso $h = 0.1$.

$$y' = \ln(x + y + 1), y(0) = 0.$$

- $f(x, y) = \ln(x + y + 1)$
- $f_x = \frac{1}{x+y+1}$
- $f_y = \frac{1}{x+y+1}$
- $y'' = f_x + f_y \cdot f = \frac{1}{x+y+1} + \frac{1}{x+y+1} \cdot \ln(x + y + 1) = \frac{1+\ln(x+y+1)}{x+y+1}$

$Para\ la\ tercera\ derivada$:

- $\frac{\partial y''}{\partial x} = -\frac{\ln(x+y+1)}{(x+y+1)^2}$
- $\frac{\partial y''}{\partial y} = -\frac{\ln(x+y+1)}{(x+y+1)^2}$
- $y''' = \frac{\partial y''}{\partial x} + \frac{\partial y''}{\partial y} \cdot y' = -\frac{\ln(d)}{d^2} - \frac{\ln(d)}{d^2} \cdot \ln(d)\,, donded = x + y + 1$

$$Fórmula\ de\ Taylor\ (orden\ 3):$$

- $y_{n+1} = y_n + h \cdot f + \frac{h^2}{2} \cdot y'' + \frac{h^3}{6} \cdot y'''$

Paso 0: $x_0 = 0, y_0 = 0$

- $f_0 = \ln(1) = 0$
- $y_0'' = \frac{1+0}{1} = 1$
- $y_0''' = 0 \quad$ (porque $\ln 1 = 0$)
- $y_1 = 0 + 0.1 \cdot 0 + \frac{0.01}{2} \cdot 1 + 0 = 0.005$

$Paso$ 1: $x_1 = 0.1, y_1 = 0.005$

- $d = 0.1 + 0.005 + 1 = 1.105$
- $f_1 = \ln(1.105) \approx 0.10017$
- $y_1'' = \frac{1+0.10017}{1.105} \approx 0.994$
- $y_1''' \approx -0.090 \quad$ (valor numérico)
- $y_2 = 0.005 + 0.1 \cdot 0.10017 + \frac{0.01}{2} \cdot 0.994 + \frac{0.001}{6}(-0.090)$
- ≈ 0.01995

$Paso\ 2\!:\ x_2 = 0.2, y_2 \approx 0.0199$

- $d = 0.2 + 0.01995 + 1 = 1.21995$
- $f_2 = \ln(1.21995) \approx 0.1989$
- $y_2^{''} \approx 0.985$
- $y_2^{'''} \approx -0.135$
- $y_3 \approx 0.01995 + 0.1 \cdot 0.1989 + \frac{0.01}{2} \cdot 0.985 + \frac{0.001}{6}(-0.135)$
- ≈ 0.04471

$$\rightarrow y(0.3) \approx 0.04471$$

7. Utilizar el método de Taylor de orden 2 para encontrar una estimación de $y(0.3)$, usando $h = 0.15$.

- $f(x, y) = x^2 + y^2$
- $f_x = 2x$
- $f_y = 2y$
- $y'' = f_x + f_y \cdot f = 2x + 2y(x^2 + y^2)$

$$Fórmula\ de\ Taylor\ (orden\ 2):$$

- $y_{n+1} = y_n + h \cdot f + \frac{h^2}{2} \cdot y''$

$Paso\ 0$: $x_0 = 0, y_0 = 0$

- $f_0 = 0$
- $y_0'' = 0$
- $y_1 = 0$

$Paso\ 1$: $x_1 = 0.15, y_1 = 0$

- $f_1 = 0.15^2 = 0.0225$
- $y_1'' = 2 \cdot 0.15 = 0.30$
- $y_2 = 0 + 0.15 \cdot 0.0225 + \frac{0.0225}{2} \cdot 0.30$
- $= 0.003375 + 0.003375 = 0.00675$

$$\rightarrow y(0.3) \approx 0.00675$$

8. Resuelve usando el método de Taylor de orden 2 para estimar $y(0.3)$, con paso $h = 0.1$.

$$y' = \sin(x) + y, \; y(0) = 1.$$

- $f(x, y) = \sin x + y$
- $f_x = \cos x$
- $f_y = 1$
- $y'' = f_x + f_y \cdot f = \cos x + \sin x + y$

$$Fórmula\ de\ Taylor\ (orden\ 2):$$

- $y_{n+1} = y_n + h \cdot f + \frac{h^2}{2} \cdot y''$

$Paso\ 0$: $x_0 = 0, y_0 = 1$

- $f_0 = \sin(0) + 1 = 1$
- $y_0'' = \cos(0) + \sin(0) + 1 = 2$
- $y_1 = 1 + 0.1 \cdot 1 + \frac{0.01}{2} \cdot 2 = 1.11$

$Paso\ 1$: $x_1 = 0.1, y_1 = 1.11$

- $f_1 = \sin(0.1) + 1.11 \approx 0.09983 + 1.11 = 1.20983$
- $y_1'' = \sin(0.1) + \cos(0.1) + 1.11 \approx 0.09983 + 0.99500 + 1.11 = 2.20483$
- $y_2 = 1.11 + 0.1 \cdot 1.20983 + \frac{0.01}{2} \cdot 2.20483$
- ≈ 1.24201

$Paso\ 2$: $x_2 = 0.2, y_2 \approx 1.24201$

- $f_2 = \sin(0.2) + 1.24201 \approx 0.19867 + 1.24201 = 1.44068$

- $y_2'' = \sin(0.2) + \cos(0.2) + 1.24201 \approx 0.19867 + 0.98007 + 1.24201 = 2.42075$
- $y_3 = 1.24201 + 0.1 \cdot 1.44068 + \frac{0.01}{2} \cdot 2.42075$
- ≈ 1.39818

$$\rightarrow y(0.3) \approx 1.39818$$